Student Understanding about Exponential Growth and the Richter Scale Following an Embodied Digital Simulation.

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Introduction

Students are faced with reasoning about non-linear growth and variable rates of change across a variety of content areas in science. The ability to apply both linear and nonlinear reasoning and to distinguish when to apply each type of reasoning is important in a wide variety of contexts from understanding bacterial population growth in Biology to kinematics in Physics. Reflecting this multidisciplinary aspect, student understanding of scale, proportion, and quantity are highlighted as crosscutting concepts in the Next Generation Science Standards in the U.S. (NGSS, 2013). These topics have traditionally been taught in mathematics classrooms, leaving science educators responsible for connecting these ideas to the scientific domains. However there is little consensus on how to facilitate or assess these connections (Pellegrino, 2013). Moreover, students often struggle to differentiate between linear and non-linear processes, and often inappropriately apply linear reasoning to non-linear problems (Modestou & Gagatsis, 2007; Wagenaar, 1982). In this study, we investigate the changes in student reasoning about non-linear growth within science contexts following the use of an embodied simulation. Specifically, our research question is, does participating in an embodied math simulation help students reason about exponential growth and facilitate transfer of their learning to science contexts?

Theoretical background

Student conceptions, from a constructivist perspective, are dynamically emergent structures which they actively construct from existing dynamic structures (Brown, 2014). In this view conceptual understanding is constructed from more elemental and intuitive experiential knowledge (diSessa, 2008). From this perspective, instruction should initially connect to students' prior knowledge by grounding instruction in student experience rather than formalisms such as symbolic equations and facilitate conceptual change through progressive formalizations such as modelling, analogical reasoning, or problematizing contexts, which connect a student's intuitions to formal ideas in science (Nathan, 2012; Niebert, et. al., 2012).

From an embodied cognition perspective, student conceptions are grounded in embodied intuitions (Niebert, et. al, 2012) such that the relationship between the body and the external world is central to processes of thinking and reasoning (Clark, 1998; Wilson, 2002). Research on embodied cognition finds that embodied interactions may lead to improved learning when they are aligned with a conceptual understanding (e.g., Glenberg, 2008; Goldin-Meadow, 2011). In addition gestures which are related to embodied intuitions rather than symbolic formalisms are more closely aligned with conceptual understandings (Alameh, et.al., 2016).

Technological advances in gesture recognition and mixed reality simulations are making it possible to design learning environments which allow students to connect body movement with abstract conceptual understanding. Research is beginning to focus on the ability for simulations to "cue" students to engage in bodily interactions which facilitate conceptual understanding with positive results (Lindgren, 2015). The prototype simulation environment employed in this study capitalizes upon students' embodied experience of addition and multiplication to help them make sense of exponential growth using both powers of two and ten. The simulation allows users to use their hands to simulate an object which represents a quantity (Figures 1a and 1b), and then manipulate the quantity through gestures which represent mathematical operations (Figure 1c).

Figure 1: (a) Wilfred adjusting the size of the virtual cube representing quantity using the Kinect, (b) the virtual cube in the simulation, and (c) simulation screen.

Method

Twenty-four high school freshman and sophomore students (15 male; mean age = 14.5) from the area surrounding a large Midwestern University were interviewed for this study. Participants completed semi-structured, task-based interviews consisting of three segments; a pre-assessment, followed by an exploration segment which engaged students in the embodied simulation, and finally a post-assessment.

Assessments

The pre and post assessments consisted of two scenarios which required students to think about a phenomenon involving exponential growth. The first scenario was a modified version of the grains of rice on a chessboard problem. The participants were asked to imagine themselves on a game show and had to select one of two prizes; either receiving 1,000 dollars per day for 30 days, or starting with one dollar and doubling the total every day for 30 days. Students were asked which option would give them the most money and to explain why this option would give them the most money. They were also asked to estimate how much money they would receive with each option and finally how they would calculate this amount if given the time and resources (e.g., a calculator). This question was selected to assess how students compared exponential growth to linear growth.

The second scenario asked the participants to think about changes in the amplitude of earthquakes using the Richter scale. Research in geoscience education has highlighted that students tend to have misconceptions about the nature of the Richter scale, often believing changes in amplitude follow a linear rather than a logarithmic relationship (Francek, 2013). Interviewers explained to students that each number on the Richter scale measures an amplitude which is ten times larger than the previous number. To investigate student reasoning about the nature of the Richter scale students were asked to compare the ratio between two numbers on the Richter scale (e.g., how many times larger is the amplitude of an earthquake measuring seven than an earthquake measuring three). They were then asked to compare the change in amplitude between two intervals (e.g., how does the change in amplitude between an earthquake of two to an earthquake of five compare to the change in amplitude between an earthquake of five to an earthquake of eight) and to explain their reasoning.

Exploration Segment

The exploration segment introduced students to gestures which represented the conceptual meaning of the four basic mathematical operations; addition, subtraction, multiplication, and division. These “concrete” gestures (e.g., stacking blocks for multiplication) were selected from student interviews where successful student conceptual understanding occurred (Alameh, et.al., 2016). Students manipulated a horizontal block to represent a given quantity (Figure 1a), also represented by a vertical bar on a graph (Figure 1c), to reach certain quantities. The students were then asked to use the simulation to solve several problems involving exponential growth involving doubling and base ten, similar to the assessment questions, but situated in a different context. For example, students were asked to calculate the number of rabbits present in a field after a year if the population doubled every month, or the number of algae cells in a pond after three months if the population increased by a factor of ten every week. They spent between 15 and 30 minutes completing the exploration segment and were not prompted to think about the game show or earthquake.

The interviews were video recorded and student responses on the pre and post assessments were scored according to a rubric. Students answers to the pre- and post-assessments were scored as correct or incorrect and their reasoning was scored based on categories of similar reasoning. For example, students who explained that doubling will eventually lead to more money than receiving the same amount everyday were scored as less sophisticated than those who explained that doubling will eventually result in more money because the rate at which the total grows is increasing. Two researchers independently scored each interview with an initial agreement of 88%. Following discussion, 100% agreement was reached.

Findings

The students were successful in selecting the doubling option as the option which yields the most money on the pretest as 17 students (70.8%) selected this option which was better than chance ($z = 2.04$, $p < .05$). However, students were less successful in being able to explain why this option would result in more money with students either stating that they had seen the problem before and knew the answer ($N = 5$) or simply stating that doubling will grow faster eventually ($N = 8$). Only three students discussed the shape or rate of change of exponential functions on the pre-test to explain why the doubling option results in more money. In addition, students were unable to estimate the total amount of money within four orders of magnitude (range from $1,000 to $200,000). Finally, only five students were able to explain how to use an exponential function to calculate the exact amount of money for the doubling option.

Over half of the students were correctly able to calculate the ratio between earthquakes of two different magnitudes on the pretest. However, students were less successful in comparing the difference between two intervals on the Richter scale. The majority of the students relied on the Richter scale values (a change from a magnitude two to a magnitude five is a change of three units), or the ratio between the amplitudes (the change from a two to a five is a factor of 1,000).

To assess the changes in student reasoning the percentage of students who improved in their reasoning from the pretest were calculated using only those students who were incorrect or not sophisticated in their reasoning on the pretest as the denominator. To assess whether this increase represented a significant change a 95% Agresti-Coull confidence interval was calculated (Agresti, & Coull, 1998). Since none of these intervals include zero, we conclude that a significant improvement was found. Similar results are obtained calculating percentage improvement using all students, as well as using confidence intervals for binomial proportions.

To investigate the transition in student reasoning, two consistency plots (Wittmann & Black, 2014) were created to visualize student change (Figure 2). In these plots students' answers to a question are plotted on the vertical axis, while their reasoning is plotted on the horizontal

Table 1: Number of students who answered correctly or provided sophisticated reasoning on the pre-test and the percentage of those who improved their answer or reasoning (N = 24)

Calculation tasks	Pre Correct	Improve (%)	Agresti-Coull 95% CI
Would you prefer to receive a prize which $1 doubles for 30 times or a prize which adds $1000 for 30 times?	17	4 (57%)	[25%,84%]
Explanation for selecting your option.	3	11 (52%)	[32%,72%]
How do you calculate the total in the doubling option?	5	7 (41%)	[20%,66%]
What is the ratio of amplitudes between two earthquakes?	13	7 (64%)	[34%,86%]
How does the size of two changes in amplitude (From 2 to 5 and from 5 to 8) compare to each other?	9	8 (53%)	[30%,75%]
Explanation of comparison between the two changes.	4	11 (55%)	[33%,75%]

axis. Students who maintain their reasoning are represented by squares, while students who change their reasoning are represented by circles for their pre-test and triangles for their post-test. In both problems, the plots show a trend towards the upper right hand corner indicating students, in general, improved their reasoning after interacting with the simulation. For the game show problem, all of the students who had previously indicated only that they had seen the problem before were able to give an explanation for why the doubling option results in more money. In the earthquake problem, those students who initially focused on the ratio between amplitudes on the pre-test tended to focus on the differences between the amplitudes on the post-test and recognize that changes in amplitude increase as one moves higher on the Richter scale, which is a key aspect of exponential change.

Finally, after completing the embodied simulation activities, 13 students increased their estimate of the amount of money the doubling option would pay after 30 days. However, only six of those students increased their estimate by more than one order of magnitude, and only two students gave an estimate within two orders of magnitude of the correct answer.

Conclusion

The results of this study are promising. After 15 to 30 minutes of work with the embodied simulation many students demonstrated more correct responses on problems involving exponential growth. More importantly, students demonstrated a transition towards providing more sophisticated explanations of exponential growth in the context of doubling and the Richter

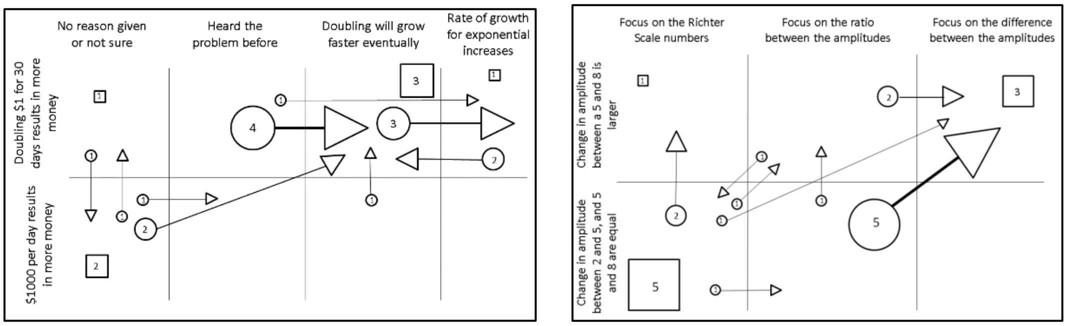

Figure 2: Change in understanding exponential growth on (a) the game show problem, and (b) Richter scale problem

scale without receiving any formal instruction on exponential functions or logarithmic scales. The alignment of the concrete embodied gestures which users adopted to interact with the simulation appear to facilitate student reasoning about exponential growth, however further study is needed to establish which features are important for facilitating student reasoning. However some incorrect conceptions were robust and may need direct instruction. For example, while students were more likely to describe a correct strategy for calculating the amount of money in the second option of the game show problem (93% compared to 68%), few students correctly applied exponential formulas after using the simulation if they had not done so before. This is unsurprising since students were not instructed on exponential equations though two students did transition to describing how to employ exponential equations to calculate the total suggesting that the simulation may be useful in concert with direct instruction or other methods. Finally, few students increased their estimates by one or more orders of magnitude. This suggests that student difficulties with orders of magnitude may underlie student understanding of exponential growth.

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