When two equals ten times one: Facilitating reasoning about exponential growth with an embodied simulation

Nitasha Mathayas, Jason Morphew, Robb Lindgren, and Sahar Alameh

University of Illinois at Urbana-Champaign

Abstract:

This study examines the effects of different facilitation strategies when students interact with an embodied simulation representing exponential growth. Twenty-two high school students were interviewed, and coded for correctness and reasoning. Overall there was a trend toward improved reasoning after students used gestures to interact with the simulation. Analysis of the individual cases revealed three strategies. First, conceptually driven questioning made students' tacit sense of growth more salient to them. Second, scaffolding emphasized this corporeal sense across contexts to encourage transfer. Third, when students become proficient in using the simulation, computational tasks can be offloaded to the simulation to engage learners in more complex concepts. An example is given for each strategy, and potential learning and design implications are described.

Introduction

Research on embodied cognition argues that the body is central to processes of thinking and reasoning (Clark, 1998; Wilson, 2002) and that designed embodied interactions may lead to improved learning (e.g. Glenberg, 2008; Goldin-Meadow, 2011). As gesture-sensing technologies become more accessible, learning environments applying principles of embodied interaction are being designed and examined for their potential to generate new concepts and new kinds of understanding (e.g. Abrahamson, 2013; Lindgren, 2016). Yet, there are still several pedagogical challenges to address with respect to practical implementation of embodied learning in subject domains such as science and mathematics. Abrahamson and Lindgren (2014) have parsed these challenges into three facets of embodied design: *activities*, *materials*, and *facilitation*. In this study, we focus on the third facet and explore how the facilitation of an embodied simulation relates to student reasoning about exponential growth. Specifically, we investigate how high school students interact with an embodied simulation representing exponential growth and how they apply these ideas within science contexts.

Theoretical background

Students engage with non-linear growth and variable rates of change across many science domains. Reflecting this multidisciplinary aspect, scale, proportion, and quantity are highlighted as crosscutting concepts in the Next Generation Science Standards in the U.S. (NGSS Lead States, 2013). However, research suggests that students often struggle to differentiate between linear and non-linear processes (Van Dooren & Greer, 2010). In particular, students often inappropriately apply linear reasoning to non-linear problems (Modestou & Gagatsis, 2007;

VanDooren, et.al., 2004; Wagenaar, 1982). While these topics are predominantly taught in mathematics classrooms, science educators are responsible for connecting these ideas to the scientific domains; however, there is little consensus or guidance on how to achieve these connections.

From a constructivist and embodied cognition perspective, students' conceptions are dynamically emergent structures evolving from existing dynamic structures (Brown, 2014), which are grounded in embodied intuitions (Niebert, March, & Treagust, 2012). Thus, instruction should facilitate conceptual change through appropriate scaffolds such as modelling, analogical reasoning, or problematizing contexts, that connect a student's intuitions to formal ideas in science (e.g. Brown, 1993, Niebert et. al, 2012, Reiser & Tabak, 2014).

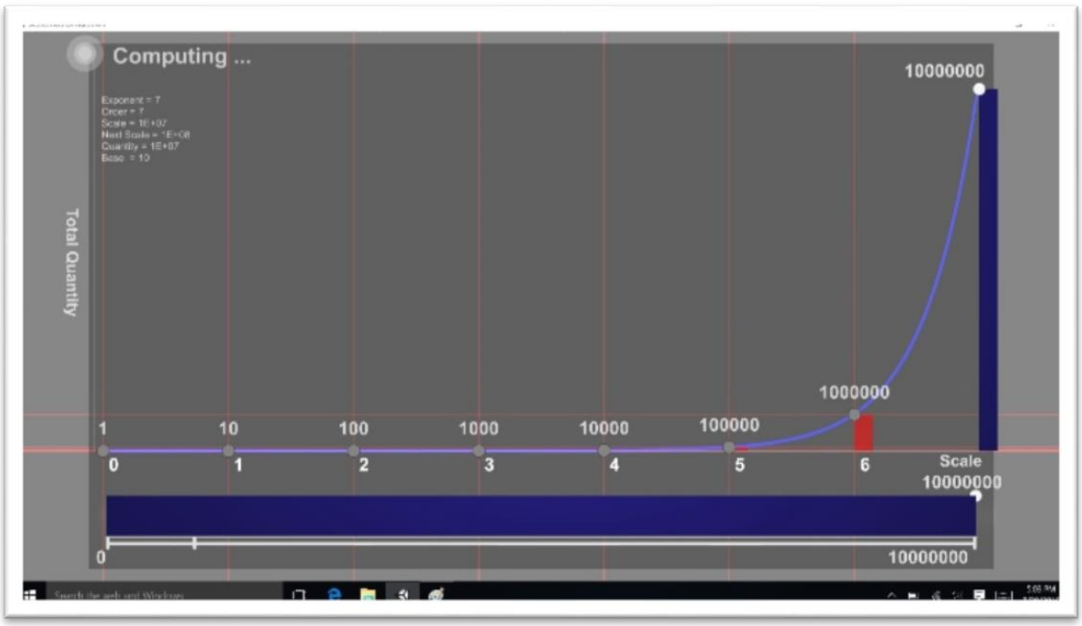

Figure 1. Simulation screen depicting exponential growth for a Base-10 scale. The x-axis represents the exponents of base 10, the y-axis represents the total quantity, and the scale below also represents the total quantity when the gesture-manipulated virtual quantity (10^6 in this screenshot) is multiplied by 10.

Our prototype simulation environment (Figure 1) capitalizes upon students' corporeal sense of linear and exponential growth (Alameh, Morphew, Mathayas, & Lindgren, 2016) to help them make sense of growth in powers of two and ten. For the design of our simulation, we target the three challenges for the design of embodied learning (Abrahamson & Lindgren, 2014). Here the simulation's *activity* draws upon the user's preexisting capacity to orient and manipulate a virtual cube (representing a quantity), that allows for embodied intuitions about how to grow that quantity to develop (Figure 2). The object's manipulation within the simulation is linked to somatic action so that the *materials* are aligned to conceptual reasoning (Figure 1). The process of engaging with exponential growth is managed by the simulation interface and the interviewers who *facilitate* user engagement by scaffolding student movement. While existing studies examine a variety scaffolds for science simulations (e.g. Fretz et. al., 2002), few investigate the real time pedagogical supports required for conceptual engagement using body movement.

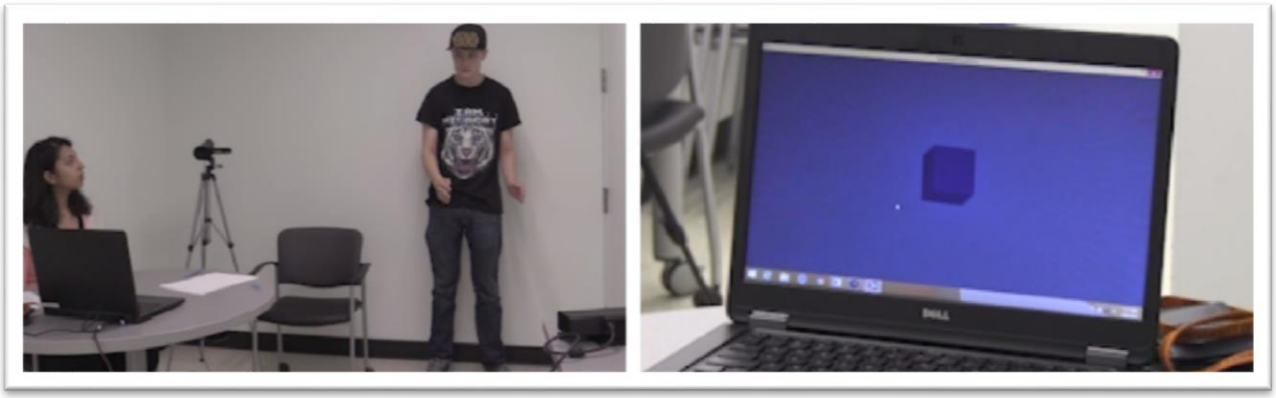

Figure 2. Wilfred adjusting the size of the virtual cube representing the quantity 1 using Microsoft's Kinect (left), and the virtual cube in the simulation (right).

Method

Twenty-two high school students (13 males and 9 females) from diverse backgrounds from the surrounding area of a large Midwestern University were interviewed for this study. Participants completed a semi-structured, task-based interview which consisted of a pre- and post-assessment along with an instructional section where students engaged with the embodied simulation and were given several tasks involving exponential growth. For example, students were asked to calculate the number of rabbits that would be present if the population doubled every month for a year, and the number of bacteria cells present in a population that increased by a factor of ten every day. Interviews were recorded and scored for correctness and reasoning (see Table 1). Two researchers independently scored each interview with an initial agreement of 84%. Following discussion, 100% agreement was reached. As seen in Table 1, several of the students improved in their thinking over the course of the interview, which is notable given the relatively short intervention (approximately 30 minutes). Students with improved reasoning scores were examined in more detail and three different facilitation strategies were identified as important for conceptual development. Space limitations prevent a comprehensive review of the data collected, but in the sections that follow, we depict each strategy with examples and a brief discussion of implications.

Findings

1. Conceptually driven questioning

One objective for this study is for students to understand changes in a quantity undergoing exponential growth. For example, the increment from 1 to 2 on a Base-10 scale is 90 units, but the increment from 2 to 3 on this scale is 900 units. While our simulation depicted this increase through the exponential graph (Figure 1), we observed that students who successfully answered increment-related questions were explicitly directed towards this difference through questioning. In the excerpt below, Emett's (Pseudonym) conception of increment changes when his attention was directed to the graph while reasoning about the population growth of rabbits.

I: So if you compare how much this bar grew from 9 to 10
S: It's growing exponentially, so the graph will be an exponential curve instead of a linear function.
I: So if we compare, how much it grew from 9 to 10 and how much it grew between 12 and 13. How do the amounts that grew compare?
S: …it doubled. So it changed from 500,000 to 10, 20- from 9 to 10 is half that from 10 to 11.
I: Okay, so the amount that it grew from 9 to 10 is half of the amount from 10 to 11.
S: Yeah
I: And so it grew more between 10 to 11 than between 9 and 10?
S: Yeah
I: Okay. So how many times bigger did it get between 9 and 10?
S: It got twice as big.
I: And how many times bigger did it get from 10 to 11?
S: Ur…it (Emett looks up as though he is realizing something) it got the same number of times bigger but it grew more.

Potential learning implications: While students are engaged with embodied simulations, asking conceptually driven questions helps them attune to subtle ideas visually depicted within the simulation, which further makes explicit their tacit understanding of growth developed through their physical interactions with the simulation.

2. Maintaining conceptual focus in multiple contexts

The simulation allows exponential growth to be situated within different scientific contexts (e.g., earthquakes, pH, or population growth). Thus, the teaching tasks asked students to use the simulation and gesture for exponential growth across multiple contexts. Students were more successful with their calculations when the interviewers brought their attention to the exponent within different contexts. For example, Ivy's (Pseudonym) attention was drawn to the exponent in four different contexts after which she adopted this strategy to reason about exponential growth. Here we share two of those four interactions where we see how Ivy steadily adopts the concept across them. In the first task about doubling the amount of money in a bank, we see that Ivy has not yet adopted the idea of the exponent. She first remembers the number of days of the money doubling and then takes a minute to recall the exponent when the interviewer prompts her again.

I: Do you remember what those numbers on the x axis represent?
S: Those were like the days right or…
I: The days sure, that's one way we could think about it
S: Um… oh! The exponent!

In the next case, when the interviewer again directs her attention to the exponent in the context of calculating the number of restaurants doubling every year, she translates the exponent to the years quite smoothly.

I: Let me show you something I didn't show you before, it's really small, but right up here right on the top here it says exponent equals 4.75. What do you think that means?
S: So in reference to that it would take 4 years and like 75…like days?…
I: Whatever .75 of a year is right?
S: Yeah

This focus on the exponent in multiple contexts is productive for Ivy as we later see that she smoothly transitions to talking about the Richter scale representing the exponents of 10 when she is questioned in the post-intervention phase.

Potential learning implications: While physical engagement with the simulation links core ideas across contexts, conceptual scaffolding by the interviewer amplifies the effect of the gesture interaction.

3. Offloading computational tasks

In our protocol, several tasks involved calculations up to large numbers to reveal the exponential trend. But when first tasked with these problems, students often attempted to mentally calculate the values. So whenever the interviewers noticed this, they encouraged the use of the simulation and asked conceptually driven questions. For example, to calculate the population of rabbits doubling, Erika initially attempted to mentally compute it but lost track of the numbers after three iterations. In the excerpt below, we see how she starts to notice the trend of the growth after she is prompted to use the simulation.

I: How many rabbits do you think we would have after a year?
S: (Doubles 12 times while watching the graph rise) Okay, that's the last one. Dear lord that's a lot of rabbits! 24,576 rabbits!
I: And we started with just 6. Well if you look at the graph now, how would you talk about how those bars change?
S: Well like the y axis, its goes up by 2 every time, multiply the number by 2, the x axis is how many times you would have to multiply the original number to get that number

I: Okay, so if we just look at how the bar changes height, does it change the same or does it change differently each time
S: It looks like it changes differently, like it's not exactly doubling each time, or it's not growing the same amount each time, it's growing somewhat bigger each time.

In this case, we see that the interviewer's encouragement to use the simulation permitted Erika to offload the tedious calculation to the simulation and notice other features of growth by doubling that she did not notice before.

Potential learning implications: Complex computations can be offloaded onto simple body actions that are paired with meaningful changes in the simulation. When students become proficient with this process, then learners can engage with deeper conceptual issues with more success.

Conclusion

This research extends previous work on embodiment and learning with new technologies by identifying three different strategies to facilitate student reasoning while engaging with an embodied simulation. Indeed, these strategies are not an exhaustive list and may be context dependent. However, we argue that when used in harmony with embodied simulations, they have the potential to generate more robust learning. While successful science learning environments benefit from synergistic scaffolding (McNeil & Krajcik, 2009; Reiser & Tabak, 2014), future work can examine the use of such scaffolds when the body becomes part of the synergy.

Table 1: Number of students who correctly answered the calculation tasks (N = 22)

Calculation tasks	Pre	Improve	Regress
Would you prefer to receive a prize which $1 doubles for 30 times or a prize which adds $1000 for 30 times?	14 (70%)	3 (15%)	1 (5%)
Explanation for selecting your option.	2 (10%)	9 (45%)	0
How would you calculate the total amount in the doubling option?	5 (25%)	5 (25%)	0
Estimate how much money you would receive with the doubling option.	0	9 (45%)	0
What is the ratio of amplitudes between two earthquakes?	10 (50%)	6 (30%)	0
How does the size of two changes in amplitude (From 2 to 5 and from 5 to 8) compare to each other?	8 (40%)	5 (25%)	3 (15%)
Explanation for the comparison between the two changes in magnitude.	3 (15%)	8 (40%)	2 (10%)

References

Abrahamson, D. (2013). Toward a taxonomy of design genres: Fostering mathematical insight via perception-based and action-based experiences. In J. P. Hourcade, E. A. Miller, & A. Egeland (Eds.), *Proceedings of the 12th Annual Interaction Design and Children Conference (IDC 2013)* (pp. 218–227). New York: The New School and Sesame Workshop.

Abrahamson, D., & Lindgren, R. (2014). Embodiment and embodied design. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (2^{nd} ed., pp. 358-376). New York: Cambridge University Press.

Alameh, S., Morphew, J. W., Mathayas, N., Lindgren, R. (2016, June). *Exploring the relationship between gesture and student reasoning regarding linear and exponential growth*. Paper presented at the International Conference of the Learning Sciences, Singapore.

Brown, D. E. (2014). Students' conceptions as dynamically emergent structures. *Science & Education, 23,* 1463 – 1483. doi: 10.1007/s11191-013-9655-9

Brown, D. E. (1993). Refocusing core intuitions: A concretizing role for analogy in conceptual change. *Journal of Research in Science Teaching, 30,* 1273–1290. doi:10.1002/tea.3660301009

Clark, A. (1998). Embodied, situated, and distributed cognition. In W. Bechtel & G. Graham (Eds.), *A companion to cognitive science* (pp. 506-517).Malden, MA: Blackwell.

Ebersbach, M., Van Dooren, W., Goudriaan, M. N., & Verschaffel, L. (2010). Discriminating Non-linearity from Linearity: Its Cognitive Foundations in Five-Year-Olds, *Mathematical Thinking and Learning, 12*. 4-19, DOI: 10.1080/10986060903465780

Fretz, E. B., Wu, H-. K., Zhang, B., Davis, E. A., Krajcik, J. S., & Soloway, E. (2002). An investigation of software scaffolds supporting modeling practices. *Research in science education, 32,* 567-589.

Glenberg, A. M. (2008). Embodiment for education. *Handbook of cognitive science: An embodied approach*, 355-372.

Goldin-Meadow, S. (2011). Learning through gesture. *Wiley Interdisciplinary Reviews: Cognitive Science, 2(6),* 595-607.

Lindgren, R. (2015). Getting into the cue: Embracing technology-facilitated body movements as a starting point for learning. In V. R. Lee (Ed.), *Learning technologies and the body: Integration and implementation in formal and informal learning environments* (pp. 39-54). New York, NY: Routledge.

McNeill, K. L., & Krajcik, J. (2009). Synergy between teacher practices and curricular scaffolds to support students in using domain-specific and domain-general knowledge in writing arguments to explain phenomena. *The journal of the learning sciences, 18,* 416-460.

Modestou, M., & Gagatsis, A. (2007) Students' Improper Proportional Reasoning: A result of the epistemological obstacle of "linearity", *Educational Psychology, 27*, 75-92, doi: 10.1080/01443410601061462

NGSS Lead States. (2013). *Next Generation Science Standards: For States, By States.* Washington, DC: The National Academies Press.

Niebert, K., Marsch, S., & Treagust, D. F. (2012). Understanding needs embodiment: A theory-guided reanalysis of the role of metaphors and analogies in understanding science. *Science Education, 96(5)*, 849-877.

Reiser, B. J., & Tabak, I. (2014). Scaffolding. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (2nd ed., pp. 44-62). New York: Cambridge University Press.

Van Dooren, W., & Greer, B. (2010). Students' behavior in linear and non-linear situations. *Mathematical thinking and learning*, *12*, 1-3.

Van Dooren, W., DeBock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2004). Remedying secondary school students' illusion of linearity: A teaching experiment aiming at conceptual change. *Learning and Instruction, 14*, 485-501.

Wagenaar, W. A. (1982). Misperception of exponential growth and the psychological magnitude of numbers. In B. Wegener (Ed.), *Social Attitudes and Psychophysical Measurement* (pp. 283–301). Hillsdale, NJ: Lawrence Erlbaum.

Wilson, M. (2002). Six views of embodied cognition. Psychonomic Bulletin & Review, 9(4), 625-636.